Analysis VI: Fläche und Integral - Lösungen

1.

a) $\int x^3\,dx=\frac{1}{4}x^4+C$

b) $\int (2x^2+4x-2)\,dx=\frac{2}{3}x^3+2x^2-2x+C$

c) $\int \frac{1}{4}\cdot(x^3-x)\,dx=\frac{1}{4}(\frac{1}{4}x^4-\frac{1}{2}x^2)+C$

d) $\int (1-x)\,dx=x-\frac{1}{2}x^2+C$

e) $\int (-\frac{1}{5}x^2+\frac{1}{2}x)\,dx=\frac{-1}{15}x^3+\frac{1}{4}x^2+C$

f) $\int (ax^2+bx+c)\,dx=\frac{1}{3}a\,x^3+\frac{1}{2}b\,x^2+cx+C$

g) $\int \frac{1}{4}\cdot(x+2)^2\,dx=\frac{1}{12}(x+2)^3+C$

h)* $\int (2x-4)^2\,dx=\frac{1}{6}(2x-4)^3+C$

2.

a) $\int (e^x+4)\,dx=e^x+4x+C$

b) $\int 2e^{2x}\,dx=e^{2x}+C$

c) $\int e^{2x+5}\,dx=\frac{1}{2}e^{2x+5}+C$

d) $\int -2e^{4x}\,dx=-\frac{1}{2}e^{4x}+C$

e) $\int 0{,}5(e^{-x}+x^2)\,dx=0{,}5(-e^{-x}+\frac{1}{3}x^3)+C$

f) $\int (1-e^{0{,}5x})\,dx=x-2e^{0{,}5x}+C$

g) $\int e^{\sin}(x)\cdot\cos(x)\,dx=e^{\sin}(x)+C$

h) $\int e^{(\ln(x)-1)}\cdot\frac{1}{x}\,dx=e^{(\ln(x)-1)}+C$

3.

a) $\int \frac{3x^2}{x^3-2}\,dx=\ln|x^3-2|+C$

b) $\int \frac{1}{x+1}\,dx=\ln|x+1|+C$

c) $\int \frac{3e^{3x}}{e^{3x}-1}\,dx=\ln|e^{3x}-1|+C$

d) $\int \frac{2x}{1+x^2}\,dx=\ln|1+x^2|+C$

e) $\int \frac{1}{3x+1}\,dx=\frac{1}{3}\ln|3x+1|+C$

4.

a) $\int sinx\,dx=-cosx+C$

b) $\int -cosx\,dx=-sinx+C$

c) $\int \sin(2x+4)\,dx=-\frac{1}{2}\cos(2x+4)+C$

d) $\int (\cos(6x)-\frac{2}{3}x)\,dx=\frac{1}{6}\sin(6x)-\frac{1}{3}x^2+C$

e) $\int \frac{cosx}{sinx}\,dx=\ln|sinx|+C$

6. Bestimmen Sie die bestimmten Integrale auf eine Dezimale genau:

a) $\int_{-2}^{0} x^3\,dx=-4$

b) $\int_{-2}^{2} \frac{1}{4}\cdot(x^3-x)\,dx=0$

c) $\int_{0}^{a} (1-x)\,dx=a-\frac{1}{2}a^2$

d) $\int_{0}^{1} (ax^2+bx+c)\,dx=\frac{1}{3}a+\frac{1}{2}b+c$

7.

a) $\int_{-1}^{0}(e^{x}+4)\,dx=4{,}63$ b) $\int_{0}^{ln2-2,5} e^{2x+5}\,dx=-72{,}21$ c) $\int_{1}^{1}-2e^{4x}\,dx=0$

d) $\int_{3}^{4}\frac{3x^2}{x^3-2}\,dx=0{,}91$ e) $\int_{0}^{1}\frac{2x}{1+x^2}\,dx=0{,}69$

8.

a) $\int_{0}^{1}x^3\,dx=\frac{1}{4}$ b) $\int_{0}^{\ln 5}e^{x}\,dx=4$ c) $\int_{-3}^{-2}\frac{1}{x}\,dx=-0{,}41$ d) $\int_{0}^{1}\frac{e^{x}}{e^{x}+1}\,dx=0{,}62$ e) $\int_{-1}^{1}\frac{2x}{1+x^2}\,dx=0$

f) $\int_{0}^{\pi}sin2x\,dx=0$ g) $\int_{0}^{1}e^{x^2}\cdot 2x\,dx=1{,}72$ h) $\int_{4}^{9}\frac{1}{2\cdot\sqrt{x}}\,dx=1$ i) $\int_{ln2}^{ln3}\frac{3e^{3x}}{e^{3x}-1}\,dx=1{,}31$

j) * $\int_{0}^{1}\frac{x}{x+1}\,dx =_{Polynomdivision} \int_{0}^{1}1-\frac{1}{x+1}\,dx=0{,}31$ k) $\int_{2}^{2}\frac{1}{2x-3}\,dx=0$

9. Bestimmen Sie u, so dass die folgende Gleichung gilt:

a) $\int_{0}^{u}x^3\,dx=30$ b) $\int_{1}^{u}e^{0.5x}\,dx=10^6$ c) $\int_{1}^{u}\frac{1}{2x+1}\,dx=10$ d) $\int_{0}^{u}\frac{2x}{1+x^2}\,dx=1$ e) $\int_{0}^{u}cos4x\,dx=\frac{1}{4}$

$u=3{,}31$ $u=26{,}24$ $u=727747792.61$ $u=\pm\sqrt{e-1}$ $u=\frac{n\cdot\pi}{4}$

10**. Entscheiden Sie, ob die folgenden unbestimmten Integrale konvergieren oder divergieren:

a) $\int_{0}^{\infty}e^{-x}\,dx=1$ b) $\int_{0}^{1}\frac{1}{x}\,dx=+\infty$ c) $\int_{0}^{1}lnx\,dx=-1$

d) $\int_{-1}^{0}\frac{x}{x+1}\,dx =_{Polynomdivision} \int_{-1}^{0}1-\frac{1}{x+1}\,dx=-\infty$ e) $\int_{1}^{\infty}\frac{1}{2\sqrt{x}}\,dx=+\infty$ f) $\int_{0}^{4}\frac{1}{(x-4)^2}\,dx=+\infty$

11. Berechnen Sie die Fläche zwischen den Funktionen f und g (berechnen Sie zuerst die Schnittpunkte)

a) $f(x)=x^2\ ;\ g(x)=3x-2$ b) $f(x)=x\cdot e^{x}\ ;\ g(x)=2x$

A = 4,17 **A = 0,094**